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D=4 N=1 Type IIB Orientifolds with Continuous Wilson Lines, Moving Branes, and their Field Theory Realization

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Abstract

We investigate four-dimensional N=1 Type IIB orientifolds with continuous Wilson lines, and their T-dual realizations as orientifolds with moving branes. When continuous Wilson lines become discrete the gauge symmetry is enhanced and the T-dual orientifold corresponds to branes sitting at the orbifold fixed points. There is a field theoretic analog describing these phenomena as D- and F-flat deformations of the T-dual model, where the branes sit at the origin (original model without Wilson lines) as well as a deformation of the T-dual model where sets of branes sit at the fixed points (the model with discrete Wilson lines). We demonstrate these phenomena for the prototype Z_3 orientifold: we present an explicit construction of the general set of continuous Wilson lines as well as their explicit field theoretic realization.

I. INTRODUCTION

Four-dimensional N=1 supersymmetric Type IIB orientifolds(see [1–8] and references therein) provide a domain of perturbative string vacua with novel properties (as opposed to the perturbative heterotic solutions) with potentially interesting phenomenological implications. One goal, that is far from being achieved, is the development of techniques that would yield a larger class of solutions than those of based on symmetric orientifold constructions. However, even within the current fairly limited class based on symmetric orbifolds, these models possess a rich structure of possible deformations which may provide a fruitful ground for further investigation of their phenomenological implications (with the ultimate goal to identify classes of models with quasi-realistic features).

The deformations in the space of supersymmetric four-dimensional solutions always have a field-theoretic realization, i.e., one identifies specific D- and F-flat directions of the original (undeformed) model. The new (deformed) supersymmetric ground states correspond to the exact string solutions, described as a power-series in the magnitude of the vacuum expectation values of the fields responsible for the deformations. In particular, one interesting phenomenon to explore is the blowing-up of the orientifold singularities [9,10], which are different in nature from that of perturbative heterotic orbifolds [11]. This phenomenon is notoriously difficult to describe within the full string theory context, since the metric of the blown-up space is not explicitly known. On the other hand the explicit field-theoretic realization in terms of (non-Abelian) flat directions allows the determination of the surviving gauge groups and massless spectrum.

Another set of deformations corresponds to the introduction of Wilson lines, both continuous and discrete, and here one expects to have, parallel with the field theory treatment, also the full string theory construction. The purpose of this paper is to address the study of such continuous Wilson lines of four-dimensional D=4 N=1 orientifolds, from both the full string theory description, i.e., by constructing explicitly these Wilson lines, and to find their T-dual interpretation, as well as from the field theory side, i.e., by identifying the moduli space of D- and F-flat directions of the effective theory; these deformations correspond on the the string side (in the T-dual picture) to the "motion" of a set of branes away from the fixed points. However, the construction of explicit continuous Wilson lines allows for an explicit string theory realization where sets of branes are located at an arbitrary distance away from the fixed points. On the other hand the field theoretical approach is only perturbative in the vacuum expectation values (VEV's), and thus in the string picture corresponds only to a deformation infinitesimally away from the undeformed model, i.e., where branes are located at the orbifold fixed points.

The purpose of this paper is to set the stage for constructions of four-dimensional N=1 Type IIB orientifolds with continuous Wilson lines, and their T-dual realizations as orientifolds with moving branes. In particular, we concentrate on explicit constructions of the continuous Wilson line solutions and the corresponding field theory realizations within the prototype, Z_3 orientifold model [1]. The explicit realization of these complementary pictures provide a beautiful correpondence between the two approaches, and sets the stage for further investigations of more involved orientifold models with continuous Wilson lines [12].

Explicit examples of discrete Wilson lines have been constructed for a number of a different orientifold models (see [4,6,7] and references therein). Continuous Wilson lines

were first addressed in [4]; however, the explicit unitary representation has not been given. The connection of models with continuous Wilson lines to the T-dual models, where branes are located away from the orbifold fixed points, while anticipated in general ([13,14,4] and references therein), was exhibited for a number of examples [4]. It is also believed that in general there should be a field theoretical realization of the same phenomena.

This paper advances these topics in several ways. In particular, we provide the first explicit unitary representation of the continuous Wilson line, specifically constructed for the Z_3 orientifold, and construct for this model the most general set of continuous Wilson lines that in the T-dual picture correspond to the moving branes. We show that these solutions allow a continuous interpolation between the original model without Wilson lines and the models with discrete Wilson lines. In addition, we provide a systematic analysis of their realization on the field theory side.

The paper is organized in the following way. In Section IIa we summarize the salient features of the Z_3 orientifold construction. In Section IIb we proceed with the construction of, first discrete and then continuous Wilson lines. When continuous Wilson lines become discrete the gauge symmetry is enhanced and the T-dual orientifold corresponds to branes sitting at the orbifold fixed points. In Section III we turn to the field theoretical analysis, by first recapitulating the techniques for a classification of D- and F-flat directions (Section IIIa). We then (Section IIIb) provide explicit constructions of such D- and F-flat directions and demonstrate their one-to-one correspondence with the continuous Wilson line string constructions.

II. Z_3 ORIENTIFOLD WITH CONTINUOUS WILSON LINES

A. $D=4$, $N=1$ Z_3 orientifold

We briefly summarize the construction [1] of four-dimensional Z_3 Type IIB orientifold models. One starts with Type IIB string theory compactified on a T^6/Z_3 (T^6-six-torus, $G_1 \equiv Z_3$ - the discrete orbifold group) and mod out by the world-sheet parity operation Ω, which is chosen to be accompanied by the same discrete symmetry $G_2 \equiv Z_3$, i.e., the orientifold group is $G = G_1 + \Omega G_2 = Z_3 + \Omega Z_3$. (Closure requires $\Omega g \Omega g' \in G_1 = Z_3$ for $g, g' \in G_2 = Z_3$.)

The compactified tori are described by complex coordinates X_i, $i = 1, 2, 3$. The action of an orbifold group Z_N on the compactified dimensions can be summarized via a twist vector $v = (v_1, v_2, v_3)$ (subject to constraint $\sum_{i=1}^{3} v_i = 1$):

$$g : X_i \to e^{2i\pi v_i} X_i \ . \tag{1}$$

For the Z_3 orientifold $v_1 = v_1 = v_3 = \frac{1}{3}$.

The tadpole cancellation, associated with the open-string modes, requires the inclusion of an even number of $D9$ branes. (The case of additional discrete symmetries in the orientifold group may require the presence of multiple sets of $D5$ branes as well.)

An open string state is denoted as as $|\Psi, ij\rangle$ where Ψ denotes the world-sheet state and i, j the Chan-Paton indices associated with the end points on a $D9$ brane. The elements $g \in G_1 = Z_3$ act on open string states as follows:

$$g : |\Psi, ij\rangle \rightarrow (\gamma_g)_{ii'} \, |g \cdot \Psi, i'j'\rangle \, (\gamma_g^{-1})_{j'j} \; . \tag{2}$$

Similarly, the elements of $\Omega G_1 = \Omega Z_3$ act as

$$\Omega g : |\Psi, ij\rangle \rightarrow (\gamma_{\Omega g})_{ii'} \, |\Omega g \cdot \Psi, j'i'\rangle \, (\gamma_{\Omega g}^{-1})_{j'j} \; , \tag{3}$$

where we have defined $\gamma_{\Omega g} = \gamma_g \gamma_\Omega$, up to a phase, in accordance with the usual rules for multiplication of group elements. Note, that Ω exchanges the Chan-Paton indices.

Since the γ_g form a projective representation of the orientifold group, consistency with group multiplication implies some conditions on the γ_g. Consider the $G_1 = G_2 = Z_3$ case; $g^3 = 1$, $\Omega^2 = 1$ and $\Omega g \Omega g' \in Z_3$ for $g, g' \in Z_3$ respectively imply:

$$\gamma_g^3 = \pm 1 \quad , \gamma_\Omega = \pm \gamma_\Omega^{\mathrm{T}} \quad , (\gamma_g^k)^* = \pm \gamma_\Omega^* \gamma_g^k \gamma_\Omega \; . \tag{4}$$

It turns out that the tadpoles cancel, if we choose the plus sign for $D9$ branes (and the minus sign for $D5$ branes) [4]. The explicit representation for the $D9$ brane sector γ_Ω is symmetric and can be chosen real:

$$\gamma_{\Omega,9} = \begin{pmatrix} 0 & 1\!\!1_{16} \\ 1\!\!1_{16} & 0 \end{pmatrix} \; , \tag{5}$$

where the subscript 9 denotes the $D9$ brane sector in which these matrices are acting.

Further, finiteness of string loop diagrams yields tadpole cancellation conditions which constrain the traces of γ_g matrices [1]

$$\mathrm{Tr}(\gamma_{Z_3}) = -4 \; . \tag{6}$$

The Z_3 twist action on the tori is given by the twist vector $v = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and its action on Chan-Paton matrices is generated by:

$$\gamma_{Z_3} = \mathrm{diag}\left(\omega 1\!\!1_{12}, 1\!\!1_4, \omega^2 1\!\!1_{12}, 1\!\!1_4\right) \; , \quad \text{where} \quad \omega = \mathrm{e}^{2\pi i/3} \; . \tag{7}$$

This choice satisfies eqs. (4) and (6). Open string states, whose Chan-Paton matrices will be denoted by $\lambda^{(i)}$, $i = 0, \ldots, 3$ in the following, give rise to space-time gauge bosons ($i = 0$) and matter states ($i = 1, 2, 3$).

Gauge bosons in the $D9$ brane sector arise from open strings beginning and ending on $D9$ branes. Invariance of these states under the action of the orientifold group requires

$$\lambda^{(0)} = -\gamma_{\Omega,9} {\lambda^{(0)}}^{\mathrm{T}} \gamma_{\Omega,9}^{-1} \qquad \text{and} \qquad \lambda^{(0)} = \gamma_{g,9} \lambda^{(0)} \gamma_{g,9}^{-1} \; . \tag{8}$$

With eq. (5) the first constraint implies that the $\lambda^{(0)}$ are SO(32) generators, while the constraints from the $\gamma_{g,9}$ will further reduce the group.

The result is the gauge group:

$$U(12) \times SO(8) \; . \tag{9}$$

The Chan-Paton matrices of the matter states have to be invariant under the action of the orientifold group as well. However, since the string vertices for the chiral matter superfields

involve the oscillator modes of the target space toroidal coordinates X_i, the Chan-Paton matrices now transform under the orbifold action (in order to render the physical states invariant under the orbifold action), thus implying:

$$\lambda^{(i)} = -\gamma_{\Omega,9}{\lambda^{(i)}}^{\rm T}\gamma_{\Omega,9}^{-1} \qquad \text{and} \qquad \lambda^{(i)} = {\rm e}^{2i\pi v_i}\gamma_{g,9}\lambda^{(i)}\gamma_{g,9}^{-1} \ . \tag{10}$$

For Z_3 orientifold this yields a matter content of three copies of

$$\psi^\alpha = (\overline{12},8)_{-1}, \quad \chi^\alpha = (66,1)_{+2} \ \ , \ \ \alpha=1,2,3. \tag{11}$$

where the subscript refers to the $U(1)$ charge of U(12). The closed string sector yields the gravity supermultiplet and the 36 (chiral) supermultiplets corresponding to the 9 untwisted ("toroidal") and 27 twisted (blowing-up) sector moduli. The moduli are gauge singlets, whose real and imaginary components arise from the NS-NS and R-R sector, respectively.

The renormalizable superpotential is of the form

$$\mathcal{W} \sim \epsilon_{\alpha\beta\gamma}\psi_i^{\alpha a}\psi_i^{\beta b}\chi^\gamma_{[a,b]}, \tag{12}$$

where α,β,γ are family indices, $\{a,b\}$-$U(12)$ indices, and i-$SO(8)$ indices.

B. Wilson Lines

Discrete Wilson Line When the action of the Wilson line on the Chan-Paton matrices, which is represented by a matrix γ_W, is such that it commutes with γ_{Z_3}, it depends only on discrete values of parameters, i.e., it describes a discrete Wilson line. Let us focus on a Wilson line, acting along the two-torus coordinate X_i, say $i=3$. It satisfies the following algebraic consistency conditions:

$$\left(\gamma_{Z_3}\gamma_W\right)^3 = +1 \ , \ [\gamma_{Z_3},\gamma_W] = 0 \ . \tag{13}$$

Further, tadpole cancellations require

$${\rm Tr}\left(\gamma_{Z_3}\right) = {\rm Tr}\left(\gamma_{Z_3}\gamma_W\right) = {\rm Tr}\left(\gamma_{Z_3}\gamma_W^2\right) = -4 \ , \tag{14}$$

To simplify the notation, let us rearrange the entries in the γ_{Z_3} matrix (7) as follows:

$$\gamma_{Z_3} = {\rm diag}\left(\omega^2 1\!\!1_{6-n}, \omega 1\!\!1_{6-n}, 1\!\!1_{4-n}, \mathbf{Z}\otimes 1\!\!1_n; \omega 1\!\!1_{6-n}, \omega^2 1\!\!1_{6-n}, 1\!\!1_{4-n}, \mathbf{Z}^*\otimes 1\!\!1_n\right) \ ; \tag{15}$$

$$\mathbf{Z} = {\rm diag}\left(\omega^2,\omega,1\right), \qquad \omega = {\rm e}^{2\pi i/3} \ , \ n=\{0,\cdots,4\} \ . \tag{16}$$

The above consistency conditions reduce to the unique solution of the discrete Wilson line:

$$\gamma_W = {\rm diag}\left(1\!\!1_{16-3n}, \omega 1\!\!1_{3n}; 1\!\!1_{16-3n}, \omega^2 1\!\!1_{3n}\right) \ . \tag{17}$$

The surviving gauge symmetry is determined by a projection:

$$\lambda^{(0)} = \gamma_W\lambda^{(0)}\gamma_W^{-1} \ , \tag{18}$$

which further breaks the gauge group down to:

$$U(12-2n)\times SO(8-2n)\times U(n)^3\ . \tag{19}$$

The matter representation is determined by the condition:

$$\lambda^{(i)}=\gamma_W\lambda^{(i)}\gamma_W^{-1}\ . \tag{20}$$

The matter comes in three copies and has the following representation (the subscripts correspond to the self-evident $U(1)$ charges):

$$\chi^\alpha=((6-n)(11-2n),1,1,1)_{-2}\ , \tag{21}$$
$$\psi^\alpha=(\overline{12-2n},8-2n,1,1,1)_1\ , \tag{22}$$
$$S^\alpha=(1,1,n,1,\bar{n})_{1,-1}\ , \tag{23}$$
$$P^\alpha=(1,1,\bar{n},n,1)_{-1,1}\ , \tag{24}$$
$$Q^\alpha=(1,1,1,\bar{n},n)_{-1,1}\ ,\quad \alpha=1,2,3\ . \tag{25}$$

The superpotential is of the form:

$$\mathcal{W}\sim\epsilon_{\alpha\beta\gamma}\psi_i^{\alpha a}\psi_i^{\beta b}\chi^\gamma_{[a,b]}+S^{\alpha i_3}_{i_1}P^{\beta i_1}_{i_2}Q^{\gamma i_2}_{i_3}\ ,\quad \text{with } \alpha\neq\beta\neq\gamma, \tag{26}$$

where α,β,γ are again family indices, $\{a,b\}$-$U(12-2n)$ indices, i-$SO(8-2n)$ indices, and $\{i_{1,2,3}\}$-respective indices for the three $U(n)$ factors.

There is a T-dual interpretation of this solution. Namely, T-dualizing the original model along, say, the third complex direction X_3, corresponds to the model with 32 $D7$ branes sitting at the origin of the third complex plane. The action of discrete Wilson lines implies that one can take n sets ($n=1,\cdots,4$) of branes (each set containing six $D7$ branes) to be at one of the two Z_3 orbifold fixed points, located away from the origin. [This motion can be accomplished in sets of six $D7$ branes; of six since each such set is moded out by six elements of the combined Z_3 and Ω group action.] Note that the string states associated with branes located at one Z_3 orbifold fixed point, away from the origin (in a particular complex plane), are related, by orientifold projection, to complex conjugate states of branes located at the other fixed point away from the origin.

Continuous Wilson Lines As the next step we generalize the construction to the case of continuous Wilson lines. We are still after a unitary representation γ_W that satisfies conditions (13)-(14), except that now it need not commute with γ_{Z_3}.

For convenience, we again rearrange the entries in the γ_{Z_3} matrix,

$$\gamma_{Z_3}=\text{diag}\left(\omega^2\mathbb{1}_{6-n},\omega\mathbb{1}_{6-n},\mathbb{1}_{4-n},\mathbb{1}_n\otimes\mathbf{Z};\omega\mathbb{1}_{6-n},\omega^2\mathbb{1}_{6-n},\mathbb{1}_{4-n},\mathbb{1}_n\otimes\mathbf{Z}^*\right)\ . \tag{27}$$

The Ansatz for the Wilson line is taken to be of the form:

$$\gamma_W=\text{diag}\left(\mathbb{1}_{16-3n},\gamma_W^0;\mathbb{1}_{16-3n},(\gamma_W^0)^*\right), \tag{28}$$

where γ_W^0 is a $(3n\times 3n)$-dimensional unitary matrix. Such a Wilson line can be uniquely determined up to unitary transformations that commute with γ_{Z_3}. The part of such unitary

transformations that affects γ_W^0 is of the form: $U_n \otimes U_3^0$, where U_n is a general $(n \times n)$-dimensional unitary matrix and U_3^0 is a (3×3)-dimensional diagonal unitary matrix. Employing such transformations in turn enables one to cast γ_W in the following most general form:

$$\gamma_W = \mathrm{diag}\left(\mathbb{1}_{16-3n}, W_1, \cdots, W_n; \mathbb{1}_{16-3n}, W_1^*, \cdots, W_n^*\right) \ , \tag{29}$$

where W_i are (3×3)-dimensional unitary matrices subject to the following consistency conditions:

$$\mathrm{Tr}(\mathbf{Z}W_i) = \mathrm{Tr}(\mathbf{Z}W_i^2) = 0 \ , \quad (\mathbf{Z}W_i)^3 = \mathbb{1}_3 \ . \tag{30}$$

We have reduced the problem to finding an explicit representation of the matrices W_i. Starting with a general Ansatz:

$$W = \begin{pmatrix} w_1 & a & b \\ a' & w_2 & c \\ b' & c' & w_3 \end{pmatrix} \quad , \tag{31}$$

the conditions (30) yield the following general form:

$$W_0 = \begin{pmatrix} w & a & b \\ a' & w+x & c \\ \frac{aa'+\omega w x}{b} & \frac{aa'-\omega^2 x(x+w)}{c} & w-\omega^2 x \end{pmatrix} \quad , \tag{32}$$

where a, b, c, w, x are complex numbers subject to the constraint det $(W) = 1$. However, unitarity imposes an immediate constraint $x = 0$; this is due to the fact that the co-factors of the diagonal entries in the above matrix are all equal to $w(w+x) - aa'$, and due to unitarity they should be proportional to the complex conjugate diagonal entries in (32).

The unitarity conditions (equating $W^\dagger$ entries with the corresponding co-factors of W) allow one to further constrain the parameters:

$$|c| = |a|, \quad |b| = |a'|, \quad |w| = \sqrt{1 - |a|^2 - |a'|^2} \ , \tag{33}$$

At this point, for the sake of simplicity, we shall change the notation $(|a|, |a'|, |w|) \to (a, a', w) \geq 0$, subject to the constraint $w = \sqrt{1 - a^2 - a'^2}$.

To further determine the phases $\phi_a, \phi_b, \phi_c, \phi_{a'}, \phi_w$, we introduce

$$\phi_A = 2\phi_{a'} + \phi_a + \phi_b - \phi_c \ , \tag{34}$$
$$\phi_B = 2\phi_a + \phi_{a'} - \phi_b + \phi_c \ , \tag{35}$$
$$\phi = \phi_w + \phi_a + \phi_{a'} \ , \tag{36}$$

and reduce the remaining unitarity conditions to the following set of equations:

$$-aa'we^{i\phi} + a'^3 e^{i\phi_A} = a'^2 \ , \tag{37}$$
$$-aa'we^{i\phi} + a^3 e^{i\phi_B} = a^2 \ , \tag{38}$$
$$-aa'we^{i\phi} + w^3 e^{3i\phi_w} = w^2 \ . \tag{39}$$

$[(\mathbf{Z}W)^3 = \mathbb{1}_3$ is automatically satisfied, provided (37)-(39) hold; namely, (37)+(38)+(39) is precisely the condition det $(W) = 1$.] The solution of (37)-(39) gives:

$$\cos(\phi_A) = \frac{(a^4 + a^2a'^2 + a'^4 + a'^2 - a^2)}{2a'^3} \,, \tag{40}$$

$$\cos(\phi_B) = \frac{(a^4 + a^2a'^2 + a'^4 - a'^2 + a^2)}{2a^3} \,, \tag{41}$$

$$\cos(3\phi_w) = \frac{(w^4 + w^2 - a^2a'^2)}{2w^3} \,, \tag{42}$$

as well as

$$\cos(\phi) = \frac{(a^4 + a^2a'^2 + a'^4 - a'^2 - a^2)}{2aa'w} \,. \tag{43}$$

Two additional constraints of eqs. (37)-(39) turn out to be automatically satisfied. In addition, the phases ϕ_A, ϕ_B, ϕ_w and ϕ are not independent, i.e.,

$$3\phi = 3\phi_w + \phi_A + \phi_B. \tag{44}$$

One can show that the expressions (40)-(43) indeed ensure $\cos(3\phi) = \cos(3\phi_w + \phi_A + \phi_B)$. (In proving this identity, the relationships: $a'^3 \sin(\phi_A) = a^3 \sin(\phi_B) = w^3 \sin(3\phi_w) = a\, a'\, w \sin(\phi)$ that follow from (37)-(39) are useful.)

Thus we have arrived at the following form of the matrix W:

$$W = \begin{pmatrix} w\, e^{i\phi_w} & a\, e^{i\phi_a} & a' e^{i\phi_b} \\ a'\, e^{i\phi_{a'}} & w\, e^{i\phi_w} & a\, e^{i\phi_c} \\ a\, e^{i(\phi_a+\phi_{a'}-\phi_b)} & a'\, e^{i(\phi_a+\phi_{a'}-\phi_c)} & w\, e^{i\phi_w} \end{pmatrix} \,. \tag{45}$$

It is determined up to diagonal unitary transformations $U_3^0 = \mathrm{diag}(e^{i\varphi_1}, e^{i\varphi_2}, e^{i\varphi_3})$, which is the most general unitary matrix that commutes with $\mathbf{Z}$, and thus provides an equivalency class for the Wilson line representations. Thus, two phase parameters in W can be "gauged away", and the only remaining phases are the two "gauge invariant" phase parameters $\phi_{A,B}$ and the phase of the diagonal element ϕ_w, all specified, by eqs. (40)-(42), in terms of two real parameters, up to signs and multiples of 2π. The latter are further constrained by (37)-(39) and (44).

To summarize, the final form of (45) is specified in terms of three real, positive parameters w, a, a', subject to the constraint $w^2 + a^2 + a'^2 = 1$. [Equivalently the Wilson line can be specified in terms of the two Euler angles φ and ψ of the three-sphere, introduced as: $w = \cos\psi$, $\quad a = \sin\psi\cos\varphi$, $\quad a' = \sin\psi\sin\varphi$.]

To study the monodromy properties of this Wilson line, it is convenient to solve (42) for $\cos\phi_w$, which has three roots:

$$\cos(\phi_w)_1 = \tfrac{1}{2}\left(e^{i\phi_0} + e^{-i\phi_0}\right) \,, \tag{46}$$

$$\cos(\phi_w)_2 = \tfrac{1}{2}\left(\omega e^{i\phi_0} + \omega^2 e^{-i\phi_0}\right) \,, \tag{47}$$

$$\cos(\phi_w)_3 = \tfrac{1}{2}\left(\omega^2 e^{i\phi_0} + \omega e^{-i\phi_0}\right) \,, \tag{48}$$

where

$$e^{i\phi_0} \equiv (A + i\sqrt{1-A^2})^{1/3} \ , \quad A \equiv \frac{(w^4 + w^2 - a^2 a'^2)}{2w^3} \ . \tag{49}$$

For the limit $A = 1$, i.e., $w = 1$, $a = a' = 0$, these solutions reduce to $\phi_w = 0, \omega, \omega^2$, which are respectively the case of no Wilson line, or discrete Wilson lines, corresponding (in the T-dual picture) to the set of $D7$ branes sitting at the origin or at one of the two fixed points. The continuous Wilson line interpolates continuously between these limits. For that purpose one has to find a path in the space of w, a, a' (or equivalently the Euler angles φ and ψ) in which $3\phi_0$ varies from 0 to $\pm 2\pi$. It is straightforward to find such paths. In particular, $3\phi_0 = 0$ and $3\phi_0 = \pi$ correspond to $A = +1$ and -1, respectively, which occur on the boundaries of the allowed region at the points $a = a' = 0$ and $a = a' = \frac{2}{3}$. There is a continuous path between these, which passes $3\phi_0 = \frac{\pi}{2}$ ($A = 0$) at, e.g., $a = a' = 1 - \frac{1}{\sqrt{3}}$. From $A = \pm 1$ one can always move along either branch of the square root, allowing a continuous interpolation, e.g., from $3\phi_0 = 0$ to $3\phi_0 = \pi$ and then to $3\phi_0 = 2\pi$ and further. For example, $3\phi_w$ can start from 0 and increase first to 2π and then to 4π as one moves back and forth between $A = +1$ and $A = -1$. An examination of (37)-(44) reveals that ϕ_A and ϕ_B each decrease by π as $3\phi_w$ increases by 2π. As $3\phi_w$ passes through 2π, $\phi_{A,B}$ and ϕ must increase discontinuously by 3π and 2π, respectively, in order to preserve the signs of the angles. This occurs at $a = a' = 0$, where the phases are indeterminant, so the changes in the Wilson line parameters are continuous. It is convenient to use the freedom in U_3^0 to require $\phi_b = \phi_c$, in which case an increase of ϕ_w by $\frac{2\pi}{3}$ (or $\frac{4\pi}{3}$) is accompanied by an increase in $\phi_{a,a'}$ by the same amount. From (30) it is clear that if W is a continuous Wilson line solution, then so are ωW and $\omega^2 W$. We thus see that these solutions, related by the discrete Z_3 symmetry for fixed a and a', can actually be related by a continuous interpolation as $3\phi_w$ varies from 0 to 2π to 4π as a function of a and a'.

The surviving gauge group is generically:

$$U(12 - 2n) \times SO(8 - 2n) \times U(1)^n \ , \tag{50}$$

as long as $W_1 \neq W_2 \neq \cdots W_n$ and $(a_i, a'_i) \neq 0$ ($i = 1, \cdots, n$). However for special values of the W_i parameters additional gauge enhancement can take place. In particular, $W_1 = W_2 \ldots = W_k$ ($k \leq n$) yields the gauge group enhancement of $U(1)^n$ to $U(k) \times U(1)^{n-k}$, with an obvious generalization to two sets $W_1 = W_2$ and $W_3 = W_4$.

This general continuous Wilson line reduces to a hybrid (continuous/discrete) Wilson line if k W matrices become diagonal with elements ω (discrete Wilson lines) (or equivalently, ω^2) and the remaining $(n - k)$ W-matrices remain off-diagonal (continuous). The gauge group $U(1)^n$ now becomes enhanced to $U(k)^3 \times U(1)^{n-k}$.

The Wilson line (29) has a T-dual interpretation in terms of n sets of six $D7$ branes each moving in, say, the third complex plane away from the orbifold fixed at the origin. In particular, when a subset of k Wilson line elements become discrete, the T-dual picture corresponds to k-sets of six $D7$ branes sitting at one (of the two) orbifold fixed point away from the origin.

The above construction of the Wilson lines for the Z_3 orientifold provides a generalization of the discrete Wilson lines (with $n = 4$) discussed in [4]. The continuous Wilson line considered there corresponds to the case with $W_1 = W_2 = W_3 = W_4$. Here, we have generalized and given an explicit unitary representation of W_i in terms of two parameters.

(When one imposes the unitarity constraint on the symmetric matrix in [4], there is a relation between the magnitude and phase of the complex parameters, and the matrix becomes a special one parameter case (with $a = a'$) of the continuous Wilson described above.) In the T-dual picture the position of the branes (in the third complex plane) should be parameterized by two real parameters, and thus for the full one-to-one correspondence with the continuous Wilson line picture the Wilson line should depend on two real parameters (a and a') as well.

Multiple Continuous Wilson Lines As the last step we proceed with the construction of multiple Wilson lines. On general grounds such Wilson lines are of the form (29) with the W_i's being of the form (45). The elements in (45) are uniquely specified in terms of two real parameters, except for two phases. As discussed above, for a single Wilson line these two phases can be removed ("gauged away") by a diagonal unitary transformation U_3^0 that commutes with $\mathbf{Z}$.

For the additional Wilson lines γ_{W_J} one cannot gauge away the two phases. However, the Wilson lines have to commute:

$$[\gamma_W, \gamma_{W_J}] = 0 \ . \tag{51}$$

This condition should in principle fix the undetermined phases. We checked that this is indeed the case. We chose the first Wilson line γ_W by fixing the gauge, i.e., choosing a specific U_3^0, so that the phases for the matrices W are fixed as:

$$\phi_a = \phi_{a'} = \phi_b \ . \tag{52}$$

Then, if the phases for the corresponding matrices W_{II} in the second Wilson line $\gamma_{W_{II}}$ are chosen as

$$\phi_{c_{II}} = \phi_{a_{II}} + \phi_c - \phi_a \ , \quad \phi_{a'_{II}} = \phi_{b_{II}} \ , \tag{53}$$

the two Wilson lines commute.

One can introduce the third Wilson line $\gamma_{W_{III}}$, with the phases subject to the analogous constraint:

$$\phi_{c_{III}} = \phi_{a_{III}} + \phi_c - \phi_a \ , \quad \phi_{a'_{III}} = \phi_{b_{III}} \ . \tag{54}$$

Such a Wilson line turns out to commute both with γ_W and $\gamma_{W_{II}}$. (Of course, one still has the freedom to make an overall unitary transformation U_3^0 on all three Wilson lines.)

The additional Wilson line $\gamma_{W_{II}}$ has again a T-dual interpretation. (The introduction of additional Wilson lines does not break the generic gauge group (50).) T-dualizing the original model along two, say, the second X_2 and third X_3, complex directions corresponds to the model with 32 $D5$ branes sitting at the origin of the second and third complex plane. The action of continuous Wilson lines $\gamma_{W_{II}}$ and γ_W then corresponds to the independent motion of n sets of six $D5$ branes in the second and the third complex planes, respectively. Since each W and W_{II} are fully specified by two real parameters, these parameters are in one-to-one correspondence with the motion of the (n) sets of branes in the two complex planes. Again, when any Wilson line element becomes diagonal (discrete) the solution corresponds to a particular set of branes reaching the orbifold fixed point in the particular plane.

The T-dual model, in which one has dualized all three complex directions $X_{1,2,3}$, corresponds to the C^3/Z_3 model of 32 $D3$ branes sitting at the origin. The introduction of the third Wilson line $\gamma_{W_{III}}$, which is also uniquely specified by two real prameters for each W_{III} matrix, parameterizes the independent motion of n sets of (six) $D3$ branes along the first plane (along with the independent motions in the second and third planes, parameterized by $\gamma_{W_{II}}$ and γ_W, respectively).

III. FIELD THEORY REALIZATION

A. Classification of F- and D-flat directions

In this section we classify the F- and D-flat directions of the new supersymmetric ground states that correspond to the deformation of the original model (and whose string theoretical construction we provided in the previous Section). We also show how to deform from the discrete Wilson line solutions corresponding to n sets of branes located at the orbifold fixed points away from the origin. For that purpose we utilize the one-to-one correspondence [16] of D-flat directions with holomorphic gauge-invariant polynomials (HIP's) built out of the chiral fields in the model. The constraints of F-flatness further require that $\langle \partial W/\partial \Phi_p \rangle = 0$ and $\langle W \rangle = 0$ for all of the massless superfields Φ_p in the model. (The detailed analysis of the blown-up Z_3 orientifold was given in [9], using this technique[1].)

We first construct a gauge invariant polynomial from the non-Abelian fields, which is a sum of monomials involving the components of the fields. Then one monomial term defines a D-flat direction. Each field in the monomial will typically have the same vacuum expectation value (VEV). The D-flat constraints for both diagonal and off diagonal generators of the non-Abelian gauge group are automatically satisfied. Other flat directions, e.g., those with different phases for the VEV's of the fields in the monomial, are gauge rotations of the original monomial.

One can also consider D-flat directions with more than one independent VEV, formed as products of other HIP's. The flat directions correspond to products of monomials from each of the HIP's, each with its own VEV. (See [9] for more detailed discussion of such issues as overlapping HIP's, involving products of HIP's which have common multiplets. Here, it suffices to check D-flatness for each case.)

[1]These techniques were previously developed (see [17]) to construct the moduli space of the flat directions for models based on perturbative heterotic string models. For simplicity, the flat direction analysis in [17] considered only the non-Abelian singlet fields in the model, in which case the flat directions correspond to gauge invariant holomorphic monomials. In the present model, the D-flat directions necessarily involve non-Abelian fields due to the matter content.

B. Flat Directions corresponding to Wilson Line Solutions

Unlike the blowing-up procedure [9,10] in which the blow-up introduced Fayet-Iliopoulos terms [15] for the anomalous $U(1)$ terms, and thus the supersymmetric ground state solutions were achieved by HIP's which have non-zero $U(1)$ charges, the deformations corresponding to the continuous Wilson lines correspond to the polynomials that are gauge invariant under the "anomalous" $U(1)$ as well.

The D-flatness condition for $SO(8)$ is

$$D^I = \sum_{\alpha,a}\sum_{i,j}(\psi_i^{\alpha a\dagger}A^I_{ij}\psi_j^{\alpha a}) = 0, \tag{55}$$

where A^I are generators of the vector representation of $SO(8)$ and $I = 1,..,28$. For $U(12)$,

$$D^J = \sum_{\alpha,i}\sum_{a,b}\psi_i^{\alpha a\dagger}\hat{T}^J_{ab}\psi_i^{\alpha b} + \sum_{\alpha}\sum_{a,b,c}\left(\chi^\alpha_{[a,c]}\right)^\dagger T^J_{ab}\chi^\alpha_{[b,c]}, \tag{56}$$

where T^J ($\hat{T}^J \equiv -T^{JT}$) are the generator matrices for the fundamental (anti-fundamental) representation of $U(12)$ and $J = 1,..,144$. (In the blown-up case one must add a constant Fayet-Iliopoulos term ξ_{FI} to the D-term for the anomalous $U(1)$ [9].)

The F-flatness conditions of the original Z_3 orientifold model are

$$\epsilon_{\alpha\beta\gamma}\psi_i^{\alpha a}\psi_i^{\beta b} = 0; \qquad \epsilon_{\alpha\beta\gamma}\psi_i^{\beta b}\chi^\gamma_{[a,b]} = 0. \tag{57}$$

For the case of the original Z_3 orientifold, one can construct D- and F-flat directions from HIP's of the form $(\psi\psi\chi)(\psi\psi\chi)$, in which each factor is separately $U(12)$ invariant, and only the product is $SO(8)$, invariant, i.e.,

$$(\psi_i^{\alpha a}\psi_j^{\alpha b}\chi^\alpha_{[a,b]})(\psi_i^{\alpha c}\psi_j^{\alpha d}\chi^\alpha_{[c,d]}). \tag{58}$$

F-flatness is ensured by taking all fields from the same family, e.g., $\alpha = 3$ for definiteness. It is necessary to consider a 6^{th} order polynomial because the cubic $(\psi\psi\chi)$ vanishes for a single family index due to the symmetry [antisymmetry] in $SO(8)$ [$U(12)$] indices. A flat direction corresponds to a specific monomial in (58), e.g., to

$$(\psi_1^{31}\psi_2^{32}\chi^3_{[1,2]})(\psi_1^{31}\psi_2^{32}\chi^3_{[1,2]}). \tag{59}$$

This is just the square[2] of $\psi_1^{31}\psi_2^{32}\chi^3_{[1,2]}$, with each of the three fields having the common VEV v, and will henceforth be denoted by $(\psi_1^{31}\psi_2^{32}\chi^3_{[1,2]})^2$.

The direction (59) appears to break $SO(8)\times U(12)$ to $SO(6)\times U(10)$. In fact, there is an additional surviving $U(1)$ generated by a combination of the broken $SO(8)$ and $U(12)$

[2] It is easy to check that (58) is only D-flat for $[a,b] = [c,d]$. For example, $(\psi_1^{31}\psi_2^{32}\chi^3_{[1,2]})(\psi_1^{33}\psi_2^{34}\chi^3_{[3,4]})$ is not D-flat under $U(12)$ because, in the terminology of [9], it involves overlapping $U(12)$ polynomials.

generators. To see this, consider a concrete representation of non-Hermitian $SO(8)$ and $U(12)$ generators labeled by $I=(kl)$ and $J=(cd)$, i.e., A_{kl} and T^c_d, with matrix elements

$$(A_{kl})_{ij} = \delta_{jk}\delta_{il} - \delta_{jl}\delta_{ik}, \tag{60}$$

and

$$(T^c_d)_{ab} = \delta_{ad}\delta_{bc}, \tag{61}$$

respectively. It is then straightforward to see that the Hermitian generator F_{12}, defined by

$$F_{ij} = i\left(T^i_j - T^j_i + A_{ij}\right), \tag{62}$$

is conserved.

Several generalizations of (59) are possible. One can consider a product of flat directions of the following schematic form:

$$\prod_{i=1}^{n}(\psi\psi\chi)^2_i, \quad \text{where} \quad n=\{1,\cdots,4\} \tag{63}$$

where each factor is a sixth order polynomial analogous to (58), we have suppressed the third family index, and each factor has a non-zero VEV v_i for each field in $\psi^{2i-1}_{2i-1}\psi^{2i}_{2i}\chi_{2i-1,2i}$. (These directions are always F-flat.) For example, one can consider

$$(\psi^{31}_1\psi^{32}_2\chi^3_{[1,2]})^2(\psi^{33}_3\psi^{34}_4\chi^3_{[3,4]})^2 \tag{64}$$

for the case $n=2$, where the two sets of fields have VEV's v_1 and v_2, respectively. The direction in (63) has the generic surviving gauge group

$$U(12-2n)\times SO(8-2n)\times U(1)^n\ . \tag{65}$$

This flat direction provides a field theory realization of a motion of n sets of (six) $D7$ branes in, say, the third complex plane, away from the fixed point at the origin, and with each set at a different location. The T-dual string theory construction in terms of a generic continuous Wilson line, given by (29), was derived in the previous Section. However, note again that the field theory allows for a realization of such a motion of branes only in the neighborhood of the original fixed point, i.e., the result is valid only in the power series expansion in terms of the VEV's of the fields.

When p of the VEV's in (63) are equal[3], the gauge factor $U(1)^p$ is enhanced to $U(p)$. For example, if $v_1=v_2$ in (64), it is straightforward to show that the generators F_{12}, F_{34}, $F_{13}+F_{24}$, and $F_{14}-F_{23}$ are conserved, and form the surviving group $U(2)$. (The $U(1)$

[3]One can show that the enhanced symmetry also holds if each pair of VEV's differs by a factor of ω or ω^2. For example, in (64) the conserved $U(2)$ generators become F_{12}, F_{34}, $\tilde{F}_{13}+\tilde{F}_{24}$, and $\tilde{F}_{14}-\tilde{F}_{23}$, where $\tilde{F}_{13}\equiv i\left(\frac{v_1}{v_2}T^1_3 - \frac{v_2}{v_1}T^3_1 + A_{13}\right)$, $\frac{v_1}{v_2}=\omega$ or ω^2, and similarly for $\tilde{F}_{24}$. We have not found an analog of this freedom in the continuous Wilson line construction.

generator is $F_{12}+F_{34}$.) Most generally, the direction in which sets of p_i of the n VEV's are equal, with $\sum_i p_i = n = \{1,\cdots,4\}$, leads to

$$U(12-2n)\times SO(8-2n)\times\prod_i U(p_i)\ , \tag{66}$$

in one to one correspondence with the continuous Wilson line solutions (29) with p_i (3×3)-dimensional matrices W_i equal, which in the T-dual picture realizes the the motion of n sets of (six) $D7$ branes with groups of p_i of them at the same position, thus providing for an enhancement of gauge symmetries.

One can generalize the construction of flat directions to include fields with all three family indices. For example, each factor in (63) can be generalized to a product of three factors, one for each family with its own VEV v_i^α but the same gauge group structure. These directions are still F-flat provided one does not mix families within the same factor. These solutions provide a field theoretical realization of the motion of branes in multiple complex planes, whose T-dual string theory construction in terms of multiple continuous Wilson line solutions was given at the end of the previous Section.

On the other hand, the discrete Wilson lines (17) provided a T-dual realization in terms of n sets of (six) $D7$ branes sitting at a fixed point away from the origin, in, say, the third complex plane; the starting gauge group there is (19) with the massless particle content (21)-(25). The flat direction corresponding to the moving of one set of branes away from the fixed point (located away from the origin) is associated with the HIP

$$S_{i_1}^{\alpha i_3}P_{i_2}^{\alpha i_1}Q_{i_3}^{\alpha i_2}\to S_1^{31}P_1^{31}Q_1^{31}, \tag{67}$$

where the restriction to a single family, say, $\alpha=3$, ensures F-flatness[4]. This breaks the $U(n)^3$ symmetry down to $U(1)\times U(n-1)^3$, where the $U(1)$ generator is the diagonal sum t_1^1 of three broken $U(1)$ generators, where

$$t_j^i \equiv t_j^{(1)i}+t_j^{(2)i}+t_j^{(3)i}. \tag{68}$$

In (68) $t^{(l)},\ l=1\ldots 3$ is the generator of the l^{th} $U(n)$ factor. Similarly, a product of $(SPQ)^q$ with distinct gauge indices, such as, e.g.,

$$\left(S_1^{31}P_1^{31}Q_1^{31}\right)\left(S_2^{32}P_2^{32}Q_2^{32}\right), \tag{69}$$

generically breaks $U(n)^3$ to $U(1)^q\times U(n-q)^3$. However, for special points with equal VEV's for each of the factors[5] there is an enhanced symmetry $U(q)\times U(n-q)^3$.

[4]Each factor SPQ can again be replaced by a product of three factors, one with its own VEV for each family, provided the family indices are not mixed within a factor. This corresponds to the multiple continuous Wilson lines and in the T-dual picture to the motion of sets of branes in three complex planes.

[5] There are also enhanced symmetries for the products $(SPQ)^q$ in the case in which the VEV's

One can choose a "hybrid" flat direction composed of:

$$\prod_i (\psi\psi\chi)^{2p_i} \prod_j (SPQ)^{q_j} \ , \quad \text{where} \sum_i p_i \leq 4-n \ , \quad \sum_j q_j \leq n \ . \tag{70}$$

The surviving gauge group is

$$U(12-2n-2\sum_i p_i) \times SO(8-2n-2\sum_i p_i) \times \prod_i U(p_i) \times \prod_j U(q_j) \times U(n-\sum_j q_j)^3. \tag{71}$$

This field theory picture of course has an analogous Wilson line realization encoded in a special choices of W_i matrices, including the choice of the specific branches for the interpretation of the phases ϕ_w.

IV. CONCLUSIONS

In this paper we focused on the study of continuous Wilson lines within four-dimensional N=1 Type IIB orientifold models. We enforced unitarity and contructed the most general set of continuous Wilson lines within the original Z_3 orientifold [1] and demonstrated that these models are in one-to-one correspondence with the T-dual models, where each Wilson line has an interpretation of n sets ($n = 1, \cdots, 4$) of (six) branes moving in one (of the three) complex planes. The number of parameters of such a continuous Wilson line is in one-to-one correspondence with the parameters that specify the location in the complex plane of each set of branes. When a sub-block of the continuous Wilson line becomes discrete, i.e., the sub-block that commutes with the corresponding sub-block of the γ_{Z_3} element and depends on a discrete parameter, this corresponds in the T-dual orientifold to branes sitting at the Z_3 orbifold fixed points. The generic Wilson lines break the original gauge group $U(12) \times SO(8)$ down to $U(12-2n) \times SO(8-2n) \times U(1)^n$. A gauge enhancement takes place for special values of the Wilson line parameters, e.g., when k sub-blocks are equal then $U(1)^k$ is promoted to $U(k)$. Similarly, when the full Wilson line becomes discrete the gauge group is enhanced to $U(12-2n) \times SO(8-2n) \times U(n)^3$. The Wilson line solution continuously interpolates between the limit of no Wilson line and the discrete solutions.

We also analysed the field theoretic analog, describing the above string constructions as D- and F-flat deformations of the effective field theory of the original model as well as deformations of the models with discrete Wilson lines. The field theory describes these string solutions only in the proximity of the original models, i.e., it allows only for a power-series expansion in the vacuum expectation values of the chiral superfields, specified by the holomorphic gauge invariant polynomials that parameterize the moduli space of the supersymmetric

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of each factor have relative phases of ω or ω^2. The generators of the enhanced symmetry are generalizations of (68) in which there are corresponding phases in the coefficients of the $t^{(l)}$. For example, if the two factors in (69) have $\frac{v_1}{v_2} = \omega$ or ω^2, then $\tilde{t}_2^1 \equiv \frac{v_1}{v_2} t_2^{(1)1} + t_2^{(2)1} + \frac{v_2}{v_1} t_2^{(3)1}$, $\tilde{t}_1^2 \equiv \tilde{t}_2^{1\dagger}$, t_1^1, and t_2^2 form a conserved $U(2)$. Again, we have not found an analog on the string theory side.

deformations of the original models. We find the explicit form of the holomorphic polynomials, that are in one-to-one correspondence with the parameters of the string constructions with the continuous Wilson lines (and their T-dual interpretation), thus quantifying the correspondence between the two complementary approaches.

The work sets the stage for further investigations of models with continuous Wilson lines. In particular, the explicit construction of the continuous Wilson lines would allow one to construct not only the massless, but also the massive spectrum of the string models. The dependence of the mass spectra and the couplings on the continuous Wilson line parameters, both from the explicit string construction as well as from the (perturbative) field theory perspective, deserves further study.

Another more general direction involves a study of a general class of four-dimensional $N = 1$ Type IIB orientifold models, in order to establish the general (and precise) correspondence between the models with continuous Wilson lines, their T-dual interpretation, as well as their field theory realization [12]. In general these models contain not only $D9$ branes but also, e.g., $D5$ branes. The latter can be located at different points on a particular two-torus T^2, where they are point-like, thus allowing for even more involved models, implementing simultaneously moving branes *and* the actions of continuous Wilson lines. Investigation of these general classes of string solutions (by determining the gauge group, the mass spectra and the couplings) would shed light on the properties of a broad class of open string models within symmetric Type IIB orientifold constructions, and may in turn lead to a discovery of potentially realistic open-string solutions.

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REFERENCES

[1] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev, *Phys. Lett.* **B385** (1996) 96, `hep-th/9606169`.

[2] M. Berkooz and R.G. Leigh, *Nucl. Phys.* **B483** (1997) 187, `hep-th/9605049`.

[3] G. Zwart, *Nucl. Phys.* **B526** (1998) 378, `hep-th/9708040`; Z. Kakushadze, *Nucl. Phys.* **B512** (1998) 221, `hep-th/9704059`; Z. Kakushadze and G. Shiu, *Phys. Rev.* **D56** (1997) 3686, `hep-th/9705163`; *Nucl. Phys.* **B520** (1998) 75, `hep-th/9706051`; L.E. Ibáñez, *JHEP* **9807** (1998) 002, `hep-th/9802103`; D. O'Driscoll, `hep-th/9801114`.

[4] G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, *Nucl. Phys.* **B536** (1999) 29, `hep-th/9804026`.

[5] Z. Kakushadze and S.H.H. Tye, *Phys. Rev.* **D58** (1998) 126001, `hep-th/9806143`; G. Shiu and S.H.H. Tye, *Phys. Rev.* **D58** (1998) 106007, `hep-th/9805157`.

[6] Z. Kakushadze, *Phys. Lett.* **B434** (1998) 269, `hep-th/9804110`; *Phys. Rev.* **D58** (1998) 101901, `hep-th/9806044`; *Nucl. Phys.* **B535** (1998) 311, `hep-th/9806008`.

[7] M. Cvetič, M. Plümacher and J. Wang, *JHEP* **0004** (2000) 004, `hep-th/9911021`.

[8] G. Aldazabal, L. E. Ibanez, F. Quevedo and A. M. Uranga, `hep-th/0005067`.

[9] M. Cvetič, L. Everett, P. Langacker and J. Wang, *JHEP* **9904** (1999) 020, `hep-th/9903051`.

[10] C. Beasley, B. R. Greene, C. I. Lazaroiu and M. R. Plesser, *Nucl. Phys.* **B566** (2000) 599, `hep-th/9907186`; M. R. Douglas, B. Fiol and C. Romelsberger, `hep-th/0003263`.

[11] M. Cvetič and L. Dixon, unpublished; M. Cvetič, in Proceedings of *Superstrings, Cosmology and Composite Structures*, College Park, Maryland, March 1987, S.J. Gates and R. Mohapatra, eds. (World Scientific, Singapore, 1987), *Phys. Rev. Lett.* **59** (1987) 1795, and *Phys. Rev. Lett.* **59** (1987) 2829.

[12] M. Cvetič, P. Langacker, M. Plümacher, A Uranga and J. Wang , work in progress.

[13] J. Lykken, E. Poppitz and S. P. Trivedi, *Nucl. Phys.* **B543** (1999) 105, `hep-th/9806080`.

[14] L. Ibáñez, *JHEP* **9807** (1998) 002, `hep-th/9802103`.

[15] M.R. Douglas and G. Moore, `hep-th/9603167`.

[16] A. Luty and W. I. Taylor, *Phys. Rev.* **D53** (1996) 3399, `hep-th/9506098`.

[17] G. Cleaver, M. Cvetič, J. R. Espinosa, L. Everett, and P. Langacker, *Nucl. Phys.* **B525** (1998) 3, `hep-ph/9711178` and *Nucl. Phys.* **B545** (1999) 47, `hep-th/9805133`.